The difference of the logarithmic ordinate and the straight-line ordinate can be derived from (21) and (22) to be

$$\frac{\Delta\phi_2 - \Delta\phi_s}{C_1} = \frac{1}{2}\left(\frac{\delta h}{\Delta H}\right)^2\left[\left(\frac{t}{\tau}\right)^2 - \frac{t}{\tau}\right]. \tag{23}$$

The maximum difference occurs at $t=\tau/2$ and is $\frac{1}{8}(\delta h/\Delta H)^2$. Thus a straight line passing equidistant from the center and end points will differ $\frac{1}{16}(\delta h)^2/(\Delta H)$ from the logarithmic curve. The suppression is therefore given by

$$S = 20 \log \frac{A}{\pi d} = -20 \log \frac{\dfrac{\delta h}{\Delta H}}{\dfrac{\pi}{16}\left(\dfrac{\delta h}{\Delta H}\right)^2}$$

$$= -20 \log \frac{\pi}{16} \frac{\delta h}{\Delta H}. \tag{24}$$

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A Stability Criterion for Tunnel Diode Amplifier

Abstract—**A relatively simple stability criterion is proposed for bandpass tunnel diode amplifiers. The results predicted from this criterion agree very closely with the results obtained from the analog computer simulation of an experimental amplifier. The derived criterion is solved to determine the limitation on the diode series inductance as a function of the diode negative resistance, and the results are plotted for a wide range of diode parameters and amplifier gains.**

The successful design of a tunnel diode amplifier depends to a large extent on the proper analysis of amplifier stability. Unfortunately analytical solutions of the stability of practical amplifier circuits are usually complicated. Consequently, stability criteria which are derived for simple diode circuits are sometimes used for preliminary design of amplifiers. One such widely used criterion is that derived for a circuit consisting of a tunnel diode terminated in a pure resistance, R_0. In terms of the diode parameters this criterion is given by

$$\frac{L_S}{RC} < (R_0 + R_S) < R \tag{1}$$

where R is the magnitude of the diode negative resistance, C is the junction capacitance, and R_S and L_S are the parasitic series resistance and inductance, respectively. This criterion as well as others, referred to as optimum, such as those given by Frisch,[1] Markowski and Davidson,[2] Davidson,[3] and Smilen and Youla,[4] would, in many cases, give too optimistic results when used in the design of bandpass tunnel diode amplifiers. For example, in the case of an S-band shunt-tuned amplifier, the limiting value of L_S for stable amplification was found to be as much as one order of magnitude smaller than the value predicted by (1).

In this correspondence a stability criterion which is relatively simple and is more suitable for amplifier design is proposed. The proposed criterion differs from the other criteria in that the circuits analyzed include the shunt tuning inductance as shown in Fig. 1. The shunt inductance and the diode junction capacitance are assumed to be resonant at the amplifier midband frequency, f_0. The stability of the proposed circuit was compared with that of an experimental amplifier (see Fig. 2), by simulating the two circuits on the analog computer. The results agreed very closely as shown in Fig. 3. The proposed criterion is derived below.

The characteristic equation of the circuit of Fig. 1 can be readily shown to be given by

$$a_3p^3 + a_2p^2 + a_1p + a_0 = 0 \tag{2}$$

where

$$a_0 = R_0(R - R_S)$$
$$a_1 = L_S(-R_0) + L(R - R_0 - R_S) + R\ R_0R_SC$$
$$a_2 = L_S(R_0RC - L) + L(R\ R_0C + R\ R_SC)$$
$$a_3 = L\ L_SR\ C.$$

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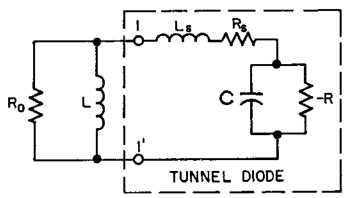

Fig. 1. Simplified amplifier circuit for stability study.

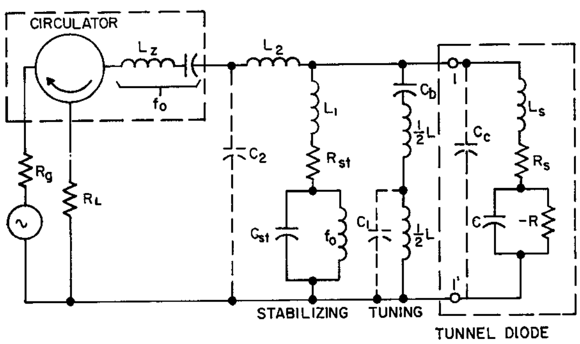

Fig. 2. Equivalent circuit of experimental tunnel diode amplifier.

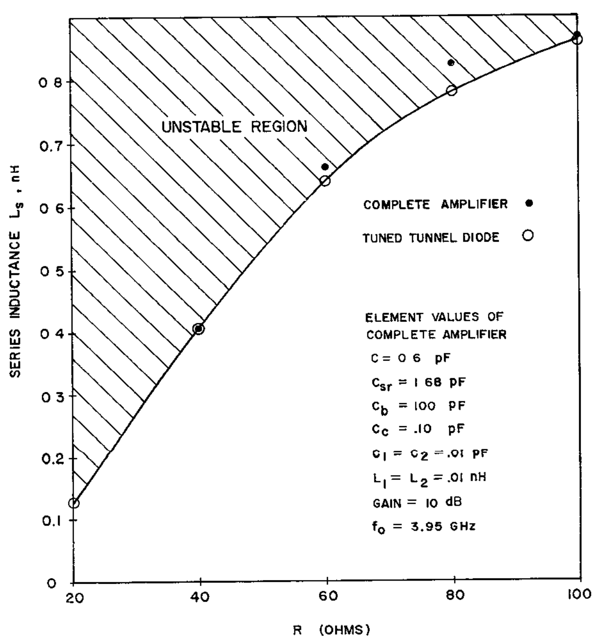

Fig. 3. Comparison of stability results obtained from the simulation of the complete amplifier circuit and proposed simplified circuit.

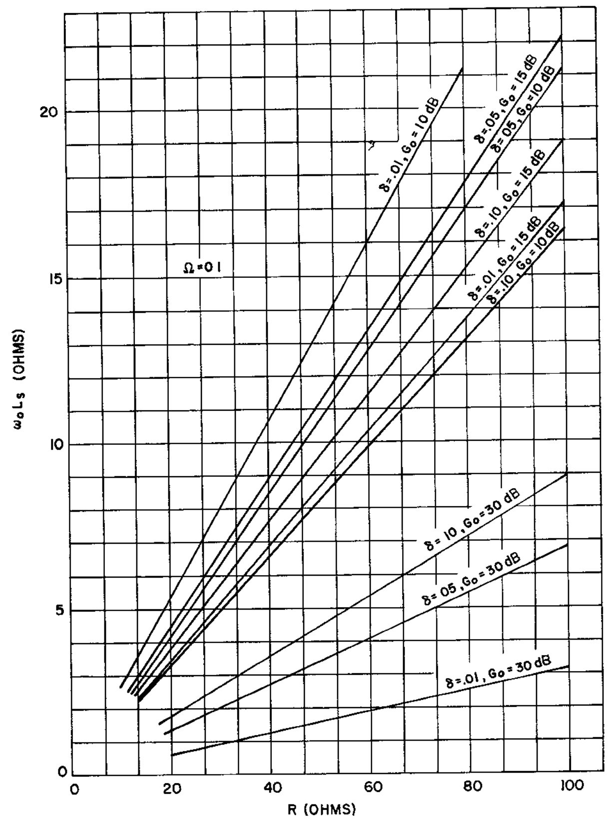

Fig. 4. Limiting series reactance for tunnel diode amplifier stability, $\Omega=0.1$.

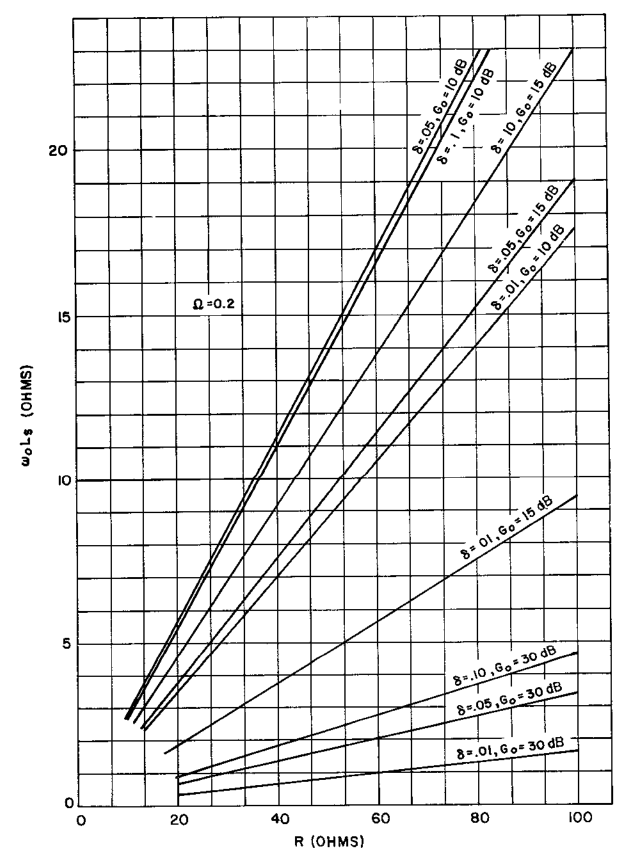

Fig. 5. Limiting series reactance for tunnel diode amplifier stability, $\Omega=0.2$.

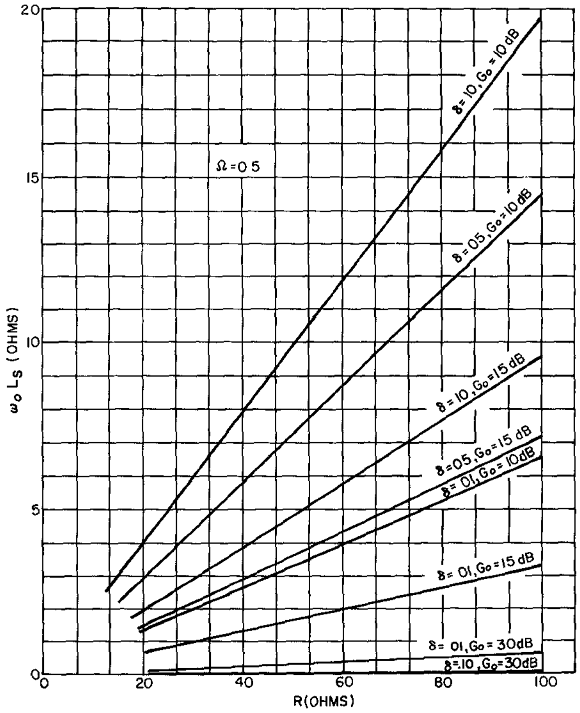

Fig. 6. Limiting series reactance for tunnel diode amplifier stability, $\Omega=0.5$.

To satisfy the Routh-Hurwitz stability criterion which requires that all roots of (2) be in the left half of the complex frequency plane p, the following conditions must be satisfied:

$$a_0 \geq 0 \qquad (3)$$

$$a_1 > 0 \qquad (4)$$

$$\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix} > 0. \qquad (5)$$

In order to analyze the stability of a particular tunnel diode amplifier using the above criterion, only the diode parameters, the source impedance R_0, and the tuning inductance L need to be known. The required value of the resistance R_0 to realize a specified gain at the midband frequency f_0 can be evaluated using the well-known gain equation

$$G_0 = 20 \log_{10} \frac{|R'| + R_0}{|R'| - R_0} \text{ dB} \qquad (6)$$

where R', the equivalent shunt negative resistance at the diode terminals, is given approximately by

$$R' = R \frac{1 - \delta(1 - \Omega^2)}{(1 - \Omega^2)},$$

where

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}},$$

$$\delta = R_S/R,$$

$$\Omega = \omega_0/\omega_c,$$

and

$$\omega_c = \frac{1}{RC}\sqrt{\frac{1-\delta}{\delta}} = \text{diode resistive cutoff frequency.}$$

The derived stability criterion can be used to determine the limiting values of diode series reactance $\omega_0 L_S$ for the stable operation of tuned amplifiers. In this case limiting values of L_S which satisfy inequalities (3) through (5) are to be determined. It was found numerically, however, that if inequality (5) is satisfied, inequalities (3) and (4) will also be satisfied over the range of interest. The numerical results for the limiting values of diode series reactance as functions of diode negative resistance are shown in Figs. 4 through 6 for a wide range of diode parameters and amplifier gain. Each figure contains curves for one value of the normalized frequency $\Omega=\omega_0/\omega_c$ and each curve is plotted for a specified gain and resistance ratio, δ.

It should be pointed out that the stability criterion derived for the tuned tunnel diode circuit is shown to be applicable to the complete circuit of an amplifier using an out-of-band stabilizing network such as that shown in Fig. 1. It is assumed that the parasitic elements are small compared with the main elements and that all tank circuits are resonant at the same frequency. These assumptions are usually valid in the case of a well-designed amplifier. In some amplifier designs, however, stabilizing networks are not used, and instead, out-of-band stability is realized by increasing the frequency selectivity of the diode circuit to be compatible with that of the circulator. The increased selectivity is achieved by adding a capacitance at the diode terminals. In this case the parasitic inductance in series with the added capacitance would have a very large effect on the amplifier out-of-band stability. In fact, this series inductance can be an order of magnitude smaller than the diode series inductance L_S and still cause instability.

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Transient Thermal Behavior of Latching Ferrite Phase Shifters

Latching ferrite phase shifters [1], [2] have been under investigation and development for a number of years as digital phase control elements in electronically steerable arrays [3]. This device employs the remanent magnetization available in a closed magnetic circuit to eliminate the large and inefficient electromagnets associated with previous ferrite phase shifters, and to provide for microsecond speed switching with low energy. The basic element of a waveguide latching phase shifter is a rectangular toroid of square hysteresis loop ferrimagnetic material with an axial magnetizing wire. Current pulses are used to switch the toroid magnetization between the two possible remanent states. The effect of changing the direction of remanent magnetization is to perturb the propagation constant of the ferrite loaded guide in the order of 10 percent, creating a differential phase shift.

From an examination of the mode of operation of this device it is seen that any effects which vary the value of remanent magnetization can change the differential phase shift. The steady-state aspects of phase variation due to RF heating and ambient temperature changes, and methods to minimize these effects have been reported by a number of investigators [4], [5]. This correspondence is concerned with the transient effects of changes in applied RF power to which this type of phase shifter could be subjected in array applications.

The geometry for thermal analysis is shown in Fig. 1. To simplify calculations we assume that RF power produces uniformly distributed heating within the body of the ferrite, that cooling occurs only by conduction in the X direction to the top and bottom waveguide walls, and that the quadrant marked A conducts one quarter of the dissipated power, ignoring the small area above and below the slot. These assumptions enable us to apply the solution of Carslaw and Jaeger [6] for transient heat flow in an internally heated infinite slab between two heat sinks.

$$\Delta T(x, t) \cong \frac{A_0 L^2}{2K} \cdot \left\{ 1 - \frac{X^2}{L^2} - \frac{32}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} \cdot \cos \frac{(2n+1)}{2L} \pi x \cdot \exp\left[-\alpha(2n+1)^2 \pi^2 \frac{t}{4L^2} \right] \right\} \qquad (1)$$

where

ΔT = temperature difference between the ferrite surface ($X=L$) and a plane in the ferrite at position X,
A_0 = heat dissipation per unit volume,
K = thermal conductivity = 0.015 cal/s $(\text{cm})^2/°\text{C}(\text{cm})$ for ferrite,
α = diffusivity = $K/\rho c$ = 0.013 cm^2/s,
ρ = density = 5.6 g/cm^3 for ferrite, and
c = specific heat = 0.2 cal/g(°C) for ferrite.

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